IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Quantum friction with a stochastic force

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1982 J. Phys. A: Math. Gen. 15 3543

(http://iopscience.iop.org/0305-4470/15/11/032)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 30/05/2010 at 15:03

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. **15** (1982) 3543–3550. Printed in Great Britain

Quantum friction with a stochastic force

R Alicki†‡ and J Messer

Sektion Physik, Theoretische Physik, Universität München, Theresienstrasse 37, D-8000 München, Federal Republic of Germany

Received 21 December 1981

Abstract. The Hamiltonian quantum theory of the damped harmonic oscillator is investigated by using an explicitly time-dependent Hamiltonian including a commutative stochastic force. The propagator is computed and shown to describe an inhomogenous quantum dynamical semigroup evolution on the Banach cone of density matrices. The moments tend to the thermal equilibrium values independently of the initial conditions, when time tends to infinity and some rescaling procedure is performed. With respect to the rescaling neither the Heisenberg nor the Schrödinger picture exists. Furthermore the energy dissipates below the zero-point energy in violation of the Heisenberg uncertainty relation.

1. Introduction

A phenomenological description of dissipative quantum systems has been achieved by the discovery of an explicitly time-dependent Hamilton function $H_t(\xi)$ by Bateman (1931), Caldirola (1941), Kanai (1948), Havas (1957) and others. This Hamiltonian defines the flow corresponding to the Langevin equation

$$m\ddot{x} + m\gamma\dot{x} + \partial V/\partial x = \xi(t), \tag{1.1}$$

where $\xi(t)$ denotes the fluctuating force due to the thermal environment of the system, and

$$H_t(\xi) = (1/2m)p^2 \, e^{-\gamma t} + (V(x, t) - x\xi(t)) \, e^{\gamma t}. \tag{1.2}$$

Whereas the Hamiltonian is only of phenomenological origin, the Langevin equation has been derived, also in the quantum case, from microscopic models. For instance Ford *et al* (1965) (see also Benguria and Kac 1981) proved (1.1) to hold for a quantum harmonic oscillator with frequency ω_0 if ξ is the Ford–Kac–Mazur quantum stochastic process, i.e.

$$\tfrac{1}{2}\langle\{\xi(t), \xi(t')\}\rangle = \frac{m\gamma}{\pi}\int_0^\infty d\omega \; \hbar\omega \cos\omega(t-t') \coth\frac{\hbar\omega}{2kT}, \tag{1.3}$$

$$[\xi(t), \xi(t')] = i\gamma\hbar(\partial/\partial t - \partial/\partial t')\delta(t-t'), \tag{1.4}$$

and the operator process $\xi(t)$ is gaussian. Here T is the temperature of the reservoir and $\langle\cdot\rangle$ is the quantum statistical expectation value. Other models (von Waldenfels

† Alexander von Humboldt fellow.
‡ On leave of absence from Institute of Theoretical Physics, Gdańsk University, Poland.

1981) with different approximations lead to (1.1) with the gaussian quantum stochastic process

$$\tfrac{1}{2}\langle\{\xi(t),\xi(t')\}\rangle = \hbar\omega_0 m\gamma \coth(\hbar\omega_0/2kT)\delta(t-t'), \tag{1.5}$$

$$[\xi(t),\xi(t')]\sim\delta(t-t'), \tag{1.6}$$

or equivalently to the master equation of Haake (1973). The quantum friction called model for (1.1) uses the Hamiltonian (1.2) but with a commutative, c-number stochastic force correlated in case of the harmonic oscillator as (1.5).

Concerning the quantum friction approach to dissipative systems, most of the considerably many publications have been reviewed by Hasse (1978), Messer (1979) and Dekker (1981). The model has been studied first by neglecting the stochastic force. It was quickly recognised that this approximation leads to unphysical consequences like the violation of the Heisenberg uncertainty relation (see e.g. Brittin 1950, Havas 1956, Kerner 1958). The significance of the (commutative) stochastic force is therefore regarded to consist in restoring the correct Heisenberg uncertainty relation. Senitzky (1960) emphasises this role of the stochastic force for the Heisenberg equation of motion (1.1), whereas Stevens (1958, 1961) and Stevens and Josephson (1959) (see also Stevens 1980) and Svin'in (1976) stress the importance of the stochastic force in the quantum friction model (1.2).

Besides stating the propagator in §§ 2 and 3 and studying its long-time asymptotic properties in § 4, we show in § 5 that even taking proper account of the commutative stochastic force in a quantum friction phenomenological model leads to inconsistencies.

2. The propagator for the damped harmonic oscillator with a stochastic force

We consider in the following the quantum damped harmonic oscillator in one dimension in the framework of the quantum friction Hamiltonian (1.2) with

$$V(x)=\tfrac{1}{2}m\omega_0^2x^2 \tag{2.1}$$

and the stochastic force ξ specified as follows: ξ is induced by a commutative (gaussian and markovian) Wiener process, i.e. ξ is a gaussian, δ-correlated c-number 'white noise',

$$\int D_0(\xi)\xi(t_1)\xi(t_2)=\langle\xi(t_1)\xi(t_2)\rangle = q\delta(t_1-t_2), \tag{2.2}$$

where $D_0(\xi)$ is a gaussian measure on the set Γ_t of trajectories of the stochastic force, $\Gamma_t=\{\xi(\tau),\tau\in[0,t]\}$. In particular, the average of the generating functional for arbitrary $A(t)$ gives

$$\left\langle \exp\left(\mathrm{i}\int_0^t A(\tau)\xi(\tau)\,\mathrm{d}\tau\right)\right\rangle = \exp\left(-\tfrac{1}{2}q\int_0^t (A(\tau))^2\,\mathrm{d}\tau\right). \tag{2.3}$$

The stochastic Hamiltonian (1.2) determines the evolution for the stochastic density matrices by the von Neumann equation

$$\dot\rho_\xi(t)=(1/\mathrm{i}\hbar)[H_t(\xi),\rho_\xi(t)]. \tag{2.4}$$

We want to describe the evolution by a propagator Λ_t on the density matrices

$\rho_t = \langle \rho_\xi(t) \rangle$, e.g. in the position representation

$$\rho_t(x, \bar{x}) = \int \Lambda_t(x, \bar{x} | x', \bar{x}') \rho_0(x', \bar{x}') \, dx' \, d\bar{x}'. \tag{2.5}$$

In the absence of the stochastic force this propagator has been found by Papadopoulos (1974). The functional integral representation of the propagator reads

$$\begin{aligned} \Lambda_t(x, \bar{x} | x', \bar{x}') &= \left\langle \int_{(x',\bar{x}')}^{(x,\bar{x})} Dx(\tau) D\bar{x}(\tau) \exp\left(\frac{i}{\hbar} \int_0^t e^{\gamma\tau} [\tfrac{1}{2} m (\dot{x}^2 - \dot{\bar{x}}^2) \right. \\ &\quad \left. - \tfrac{1}{2} m \omega_0^2 (x^2 - \bar{x}^2) + \xi(\tau)(x - \bar{x})] \, d\tau \right) \right\rangle \\ &= \int_{(x',\bar{x}')}^{(x,\bar{x})} Dx(\tau) D\bar{x}(\tau) \exp\left[\frac{i}{\hbar} \int_0^t \left(e^{\gamma\tau} [\tfrac{1}{2} m (\dot{x}^2 - \dot{\bar{x}}^2) \right. \right. \\ &\quad \left. \left. - \tfrac{1}{2} m \omega_0^2 (x^2 - \bar{x}^2)] + \frac{1}{2} \frac{i}{\hbar} q \, e^{2\gamma\tau} (x - \bar{x})^2 \right) d\tau \right] \end{aligned} \tag{2.6}$$

by using (2.3). This time evolution is equivalent to the master equation

$$\dot{\rho}_t = (1/i\hbar)[H_t(0), \rho_t] - (q/2\hbar^2) \, e^{2\gamma t} [\hat{x}, [\hat{x}, \rho_t]], \tag{2.7}$$

which defines a completely positive inhomogeneous quantum dynamical semigroup (see e.g. Lindblad 1976a, Davies 1974) on the density matrices. Here position and momentum operators are indicated explicitly by $\hat{x}$ and $\hat{p}$. The representation of general quantum dynamical semigroups by path integrals was investigated by Alicki *et al* (1981). The master equation (2.7) is equivalent to the quantum Fokker–Planck equation in the Glauber coherent state representation found by Brinati and Mizrahi (1980).

3. Representation of the propagator by Svin'in's method

In order to obtain a more convenient representation of the evolution operator Λ_t we change the variables in the path integral representation (2.6) according to Svin'in's (1976) transformation

$$y = x - \eta(t), \qquad \bar{y} = \bar{x} - \eta(t), \tag{3.1}$$

where

$$\ddot{\eta} + \gamma \dot{\eta} + \omega_0^2 \eta = \xi(t)/m \tag{3.2}$$

with initial conditions $\eta(0) = \dot{\eta}(0) = 0$. It follows immediately that

$$\Lambda_t(y, \bar{y} | y', \bar{y}') = \int D_0(\xi) \exp\left(-\frac{i}{\hbar} m \, e^{\gamma t} (y - \bar{y}) \dot{\eta} \right) K_t(y + \eta, \bar{y} + \eta | y', \bar{y}'), \tag{3.3}$$

with

$$\eta = \eta(t), \qquad \dot{\eta} = \dot{\eta}(t),$$

and where

$$K_t(z, \bar{z} | z', \bar{z}') = V_t(z, z') V_t^*(\bar{z}, \bar{z}'), \tag{3.4}$$

$$V_t = \boldsymbol{T} \exp\left(-\frac{\mathrm{i}}{\hbar}\int_0^t H_\tau(0)\,\mathrm{d}\tau\right) \tag{3.5}$$

denotes the propagator without stochastic force, which was derived using path integrals by Papadopoulos (1974).

For given $y, y', \bar{y}, \bar{y}'$ and t the function

$$G(\eta, \dot{\eta}) = \exp[-(\mathrm{i}/\hbar)m\,\mathrm{e}^{\gamma t}(y-\bar{y})\dot{\eta}]K_t(y+\eta, \bar{y}+\eta \mid y', \bar{y}') \tag{3.6}$$

is defined on $\Gamma_t = \{\xi(\tau); \tau \in [0, t]\}$. We introduce the one-to-one map T from the set Γ_t to the set Σ_t of all solutions of the equation (3.2) with the initial conditions $\eta(0) = \dot{\eta}(0) = 0$ by $T[\xi](\tau) = \eta(\tau)$. Introducing a second map $F: \Sigma_t \to \mathbb{R}^2$ by $F[\{\eta(\tau), \tau \in [0, t]\}] = (\eta(t), \dot{\eta}(t))$, one can define

$$P: \Gamma_t \to \mathbb{R}^2, \qquad P = F \circ T \tag{3.7}$$

and therefore

$$G(\eta, \dot{\eta}) = G(P[\xi]). \tag{3.8}$$

Let $\Sigma_t(\mathrm{d}\eta, \mathrm{d}\dot{\eta})$ denote the set of trajectories $\{\eta(\tau), \tau \in [0, t], \eta(t) \in [\eta, \eta+\mathrm{d}\eta], \dot{\eta}(t) \in [\dot{\eta}, \dot{\eta}+\mathrm{d}\dot{\eta}]\}$. A probability distribution $W_t(\eta, \dot{\eta})$ is then defined by

$$\begin{aligned} W_t(\eta, \dot{\eta})\,\mathrm{d}\eta\,\mathrm{d}\dot{\eta} &= D_0(T^{-1}[\Sigma_t(\mathrm{d}\eta, \mathrm{d}\dot{\eta})]) \\ &= D_0(P^{-1}([\eta, \eta+\mathrm{d}\eta] \times [\dot{\eta}, \dot{\eta}+\mathrm{d}\dot{\eta}])) \\ &=: D_0(P^{-1}(\mathrm{d}\eta \cdot \mathrm{d}\dot{\eta})). \end{aligned} \tag{3.9}$$

$W_t(\eta, \dot{\eta})$ fulfils the Fokker–Planck equation for the harmonic oscillator with initial conditions

$$W_0(\alpha, \dot{\alpha}) = \delta(\alpha)\,\delta(\dot{\alpha}), \tag{3.10}$$

and its manifest form can be found in Chandrasekhar (1943) or Wang and Uhlenbeck (1945). Using (3.3)–(3.9) we obtain

$$\begin{aligned} \Lambda_t(y, \bar{y} \mid y', \bar{y}') &= \int_{\Gamma_t} D_0(\xi) G(P[\xi]) \\ &= \int_{\mathbb{R}^2} D_0(P^{-1}(\mathrm{d}\eta \cdot \mathrm{d}\dot{\eta})) G(\eta, \dot{\eta}) \\ &= \int_{\mathbb{R}^2} \mathrm{d}\eta\,\mathrm{d}\dot{\eta}\, W_t(\eta, \dot{\eta}) \exp\left(-\frac{\mathrm{i}}{\hbar} m\,\mathrm{e}^{\gamma t}(y-\bar{y})\dot{\eta}\right) K_t(y+\eta, \bar{y}+\eta \mid y', \bar{y}'). \end{aligned} \tag{3.11}$$

For further discussions it is useful to state the following operator form of (3.11):

$$\begin{aligned} \Lambda_t \rho = \int \mathrm{d}\eta\,\mathrm{d}\dot{\eta}\, W_t(\eta, \dot{\eta}) \exp[(\mathrm{i}/\hbar)(\eta\hat{p} - m\,\mathrm{e}^{\gamma t}\dot{\eta}\hat{x})] \\ \times V_t \rho V_t^* \exp[-(\mathrm{i}/\hbar)(\eta\hat{p} - m\,\mathrm{e}^{\gamma t}\dot{\eta}\hat{x})], \end{aligned} \tag{3.12}$$

which equivalently defines the semigroup (2.7).

4. Approach to equilibrium

The large-time behaviour of the expectation values of observables $A(\hat{x})$, which depend only on the position operator $\hat{x}$, is seen by investigating the propagator $\Lambda_t(y, y|y', \bar{y}')$ for large t. From (3.11) and Chandrasekhar (1943) (or Svin'in 1976):

$$\Lambda_t(y, y|y', \bar{y}') = \int \mathrm{d}\eta \; W_t(\eta) K_t(y+\eta, y+\eta|y', \bar{y}'), \tag{4.1}$$

with

$$W_t(\eta) = (\sqrt{\pi}\sigma_t)^{-1} \exp(-\eta^2/\sigma_t^2) \tag{4.2}$$

and

$$\sigma_t^2 = \frac{q}{m^2\gamma\omega_0^2}\left[1 - \mathrm{e}^{-\gamma t}\left(1 + \frac{\gamma}{2\Omega}\sin 2\Omega t + \frac{\gamma^2}{2\Omega^2}\sin^2 \Omega t\right)\right], \tag{4.3}$$

where $\Omega^2 = \omega_0^2 - \gamma^2/4$ is taken here to be positive (weak damping case). Using the propagator K_t of Papadopoulos (1974) in (4.1) and performing the gaussian integration with respect to η, we arrive at

$$\begin{aligned}\Lambda_t(y, y|y', \bar{y}') &= \sqrt{\pi}\sigma_t N_t \exp[A(t)(y'^2 - \bar{y}'^2)](\sqrt{\pi}\sigma_t)^{-1} \\ &\quad \times \exp\left(-\frac{x^2}{\sigma_t^2}\right) \exp\left[-\left(\frac{m\Omega\,\mathrm{e}^{(\gamma/2)t}\sigma_t}{2\hbar \sin \Omega t}(y' - \bar{y}') + \frac{\mathrm{i}y}{\sigma_t}\right)^2\right]\end{aligned} \tag{4.4}$$

for some $A(t)$ and the normalisation factor N_t. In the limit $t \to \infty$ the expression (4.4) becomes

$$\Lambda_\infty(y, y|y', \bar{y}') = \delta(y' - \bar{y}')(\sqrt{\pi}\sigma_\infty)^{-1} \exp(-y^2/\sigma_\infty^2). \tag{4.5}$$

Since (see e.g. Svin'in 1976)

$$q = \hbar\omega_0 m\gamma \coth(\hbar\omega_0/2kT), \tag{4.6}$$

where T denotes the temperature of the surrounding reservoir, the expectation values of $A(\hat{x})$ tend to their equilibrium values for $t \to \infty$, for *any* initial conditions, i.e. this convergence is independent of the chosen initial preparation.

For observables which depend only on the canonical momentum $\hat{p}$, $A(\hat{p})$, the mean values can be computed, using the Fourier transform

$$\tilde{\Lambda}_t(p, p|y', \bar{y}') = (2\pi\hbar)^{-1} \int \mathrm{d}y\, \mathrm{d}\bar{y} \exp\left(-\frac{\mathrm{i}}{\hbar}p(y - \bar{y})\right)\Lambda_t(y, \bar{y}|y', \bar{y}'). \tag{4.7}$$

For large t, similar to (4.5), we obtain the equilibrium values expressed in the rescaled kinetical momentum $p_{\mathrm{kin}} = p\,\mathrm{e}^{-\gamma t}$ independently of the initial conditions.

5. Non-existence of dynamical maps

The evolution of the expectation values for observables $A(\hat{x})$ or $A(\hat{p})$ is given by

$$\omega_\rho(A(\hat{x}))(t) = \mathrm{Tr}(A(\hat{x})\Lambda_t\rho), \tag{5.1}$$

$$\omega_\rho(A(\hat{p}))(t)=\mathrm{Tr}(A(\mathrm{e}^{-\gamma t}\hat{p})\Lambda_t\rho), \tag{5.2}$$

for the initial state ρ. The reason to use the rescaled momentum is the result of § 4:

$$\lim_{t\to\infty}\omega_\rho(A(\hat{p}))(t)=\langle A(\hat{p}_{\mathrm{kin}})\rangle_{\mathrm{equ}}. \tag{5.3}$$

According to the principles of quantum mechanics the evolution (5.1), (5.2) ought to be expressed by a family of unity preserving, positive linear maps $\Lambda_t^*\Xi_t$ on the algebra of observables with $\Xi_t(A(\hat{x}))=A(\hat{x})$ and $\Xi_t(A(\hat{p}))=A(\mathrm{e}^{-\gamma t}\hat{p})$. Then $\Lambda_t^*\Xi_t$ defines the evolution in the Heisenberg picture and $\Xi_t^*\Lambda_t$ in the Schrödinger picture. Clearly Ξ_t cannot be an automorphism and an extension of Ξ_t to the non-commutative part of the algebra is ambiguous. Moreover, even on observables which are sums $A(\hat{x})+B(\hat{p})$ the map $\Lambda_t^*\Xi_t$ cannot be positive. In (3.12) let us denote $\Lambda_t^*=U_t\Phi_t$ with $U_tA=V_t^*AV_t$ acting on the observable A; then $\Phi_t\Xi_t$ should be positive. Choosing

$$A=\hat{p}^2+\alpha^2x^2-\alpha\hbar \tag{5.4}$$

for $\alpha\in\mathbb{R}$ to be specified later, we obtain

$$\Phi_t\Xi_t(A)=B_\alpha(t)+c_\alpha(t), \tag{5.5}$$

$$B_\alpha(t)=D_\alpha^*(t)D_\alpha(t), \tag{5.6}$$

$$D_\alpha(t)=\hat{p}\,\mathrm{e}^{-\gamma t}-\mathrm{i}\alpha\hat{x}, \tag{5.7}$$

$$c_\alpha(t)=-\alpha\hbar(1-\mathrm{e}^{-\gamma t})+\tfrac{1}{2}\alpha^2\sigma_t^2+\tfrac{1}{2}m^2\sigma_{\dot{\eta}}^2 \tag{5.8}$$

$$\begin{aligned}&=(1-\mathrm{e}^{-\gamma t})\left(\frac{q\alpha^2}{2m^2\omega_0^2\gamma}+\frac{q}{2\gamma}-\alpha\hbar\right)\\&\quad-\frac{q\alpha^2}{2m^2\omega_0^2\gamma}\mathrm{e}^{-\gamma t}\left(\frac{\gamma}{2\Omega}\sin 2\Omega t+\frac{\gamma^2}{2\Omega^2}\sin^2\Omega t\right)\\&\quad-\frac{q}{2\gamma}\mathrm{e}^{-\gamma t}\left(-\frac{\gamma}{2\Omega}\sin 2\Omega t+\frac{\gamma^2}{2\Omega^2}\sin^2\Omega t\right).\end{aligned} \tag{5.9}$$

Let $\alpha>q/\hbar\gamma=m\omega_0\coth(\hbar\omega_0/2kT)$; then $c_\alpha(0)=0$ but $\dot{c}_\alpha(0)<0$. There exists a $\tau>0$ such that $c_\alpha(\tau)<0$. We choose a vector state ρ_τ being the projection on the vector ψ_τ defined by $D_\alpha(\tau)\psi_\tau=0$. Consequently $\mathrm{Tr}(\Phi_\tau\Xi_\tau(A)\rho_\tau)<0$, which concludes the proof.

Of interest for physics is e.g. the energy observable

$$\Xi_t(E-E_0)=(1/2m)\hat{p}^2\,\mathrm{e}^{-2\gamma t}+\tfrac{1}{2}m\omega_0^2x^2-\tfrac{1}{2}\hbar\omega_0 \tag{5.10}$$

which corresponds to the choice $\alpha=m\omega_0=:\alpha_0$ in (5.4). If we consider as state of the oscillator the projection $U_t^*\rho_0$ on the pseudostationary ground state (Hasse 1975)

$$\psi_0(t)=V_t\psi_0(0)=N_0\exp[(\tfrac{1}{4}\gamma-\tfrac{1}{2}\mathrm{i}\Omega)t-(m/2\hbar)(\Omega+\tfrac{1}{2}\mathrm{i}\gamma)\,\mathrm{e}^{\gamma t}x^2], \tag{5.11}$$

then

$$\mathrm{Tr}((2m)^{-1}B_{\alpha_0}(t)U_t^*\rho_0)=\tfrac{1}{2}\hbar\omega_0(\omega_0/\Omega-1)\,\mathrm{e}^{-\gamma t} \tag{5.12}$$

and using (5.8), (5.9),

$$\mathrm{Tr}(\Phi_{t_0}\Xi_{t_0}(E-E_0)U^*_{t_0}\rho_0)$$
$$=\tfrac{1}{2}\hbar\omega_0\bigg[\Delta(1-\mathrm{e}^{-\gamma t_0})-\mathrm{e}^{-\gamma t_0}\frac{\gamma^2}{4\Omega^2}\coth\Big(\frac{\hbar\omega_0}{2kT}\Big)$$
$$-\mathrm{e}^{-\gamma t_0}\frac{\gamma^2}{4\Omega^2}\Delta-\mathrm{e}^{-\gamma t_0}\frac{1}{\Omega^2}(\omega_0^2-\omega_0\Omega)\bigg] \tag{5.13}$$

with $\Delta=\coth(\hbar\omega_0/2kT)-1$ and $2\Omega t_0=\pi$. Thus with this initial condition the mean energy dissipates below the zero-point energy if the temperature T of the bath is sufficiently low. This result implies immediately the invalidity of the Heisenberg uncertainty relation by taking the square of the sum and the square of the difference of the expectation values of kinetic and potential energy; thus

$$(\Delta x)^2(t_0)(\Delta p_{\mathrm{kin}})^2(t_0)=\omega_{\rho_0}(\hat{x}^2)(t_0)\omega_{\rho_0}(\hat{p}^2\,\mathrm{e}^{-2\gamma t})(t_0)<\hbar^2/4 \tag{5.14}$$

for $kT\ll\hbar\omega_0$.

6. Conclusions

The considerations of § 5 prove the non-existence of a family of dynamical maps, i.e. the non-existence of the Heisenberg and Schrödinger pictures, if we require that our model with friction-inbuilt Hamiltonian and a commutative stochastic force should describe the approach to equilibrium properly. Moreover, independent of this requirement, we showed that for sufficiently low thermal energy kT (compared with $\hbar\omega_0$) there exists a time t_0 where for some initial state the mean energy is less than the zero-point energy, i.e. $\omega_{\rho_0}(E)(t_0)<\hbar\omega_0/2$ and the uncertainty relation does not hold, i.e. $\Delta x(t_0)\Delta p_{\mathrm{kin}}(t_0)<\hbar/2$. This violation of the principles of quantum mechanics might be caused by the non-quantum nature of the stochastic force or by the artificial frictional Hamiltonian $H_t(0)$. Usually in the markovian approximation dissipative quantum systems are described by quantum dynamical semigroups which arise from microscopic models in the weak coupling or singular coupling limit (see e.g. Davis (1976) and Gorini *et al* (1978) as reviews). An example of this description of the Brownian motion of a quadratic system is given by Lindblad (1976b). If the stochastic force is treated as a quantised noise, then Gorini and Kossakowski (1976) showed rigorously for an N-level system that damping and pumping is described together by the quantum stochastic force and there is no need for the artificial Hamiltonian $H_t(0)$.

Acknowledgment

One of the authors (RA) is grateful to the Alexander von Humboldt Stiftung for a research scholarship.

References

Alicki R, Langouche F and Roekaerts D 1981 *Physica* A 111A 622
Bateman H 1931 *Phys. Rev.* **38** 815

Benguria R and Kac M 1981 *Phys. Rev. Lett,* **46** 1
Brinati J R and Mizrahi S S 1980 *J. Math. Phys.* **21** 2154
Brittin W E 1950 *Phys. Rev.* **77** 396
Caldirola P 1941 *Nuovo Cimento* **18** 393
Chandrasekhar S 1943 *Rev. Mod. Phys.* **15** 1
Davies E B 1974 *Commun. Math. Phys.* **39** 91
—— 1976 *Quantum Theory of Open Systems* (London: Academic)
Dekker H 1981 *Phys. Rep.* **80** 1
Ford G W, Kac M and Mazur P 1965 *J. Math. Phys.* **6** 504
Gorini V, Frigerio A, Verri M, Kossakowski A and Sudarshan E C G 1978 *Rep. Math. Phys.* **13** 149
Gorini V and Kossakowski A 1976 *J. Math. Phys.* **17** 1298
Haake F 1973 in *Quantum Statistics in Optics and Solid-State Physics, Springer Tracts in Modern Physics* vol 66 (Berlin: Springer) p 98
Hasse R W 1975 *J. Math. Phys.* **16** 2005
—— 1978 *Rep Prog. Phys.* **41** 1027
Havas P 1956 *Bull. Am. Phys. Soc.* **1** 337
—— 1957 *Nuovo Cimento Suppl.* **5** 363
Kanai E 1948 *Prog. Theor. Phys.* **3** 440
Kerner E H 1958 *Can. J. Phys.* **36** 371
Lindblad G 1976a *Commun. Math. Phys.* **48** 119
—— 1976b *Rep. Math. Phys.* **10** 393
Messer J 1979 *Acta Phys. Austr.* **50** 75
Papadopoulos G J 1974 *J. Phys. A: Math., Nucl. Gen.* **7** 209
Senitzky I R 1960 *Phys. Rev.* **119** 670
Stevens K W H 1958 *Proc. Phys. Soc.* **72** 1027
—— 1961 *Proc. Phys. Soc.* **77** 515
—— 1980 *Phys. Lett.* **75A** 463
Stevens K W H, and Josephson B 1959 *Proc. Phys. Soc.* **74** 561
Svin'in I R 1976 *Teor. Mat. Fiz.* **27** 270
von Waldenfels W 1981 *Private communication*
Wang M C and Uhlenbeck G E 1945 *Rev. Mod. Phys.* **17** 323